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NOTE

A Study of Stationary, Axially Symmetric Space-time Geometries Satisfying Modified Double Duality Equations Using the Exterior Calculus Package X^TR for REDUCE

1. INTRODUCTION

Various types of gravitational theories that generalize Einstein's theory were being discussed during the recent years. A generalization that received a lot of attention was due to Stephenson, Kilmister, Yang, and several others [1–3]. In this theory the gravitational action is written in analogy with Yang–Mills type gauge theories, so that the field equations are obtained by varying a quadratic curvature invariant. In a perturbative approach to field quantisation this theory leads to a renormalisable, albeit in general, non-unitary quantum gravity [4]. Even at the classical level the theory has some serious problems. Namely, it admits non-physical solutions [5, 6]. Therefore, in an attempt to constrain the quadratic theory further it was suggested to add on the Einstein–Hilbert action [7]. If it is further allowed to add a cosmological constant, then some remarkable simplifications follow. For a definite value of the cosmological constant, there is a set of modified double dual curvature equations whose integrability conditions give precisely the variational field equations [8, 9]. Thus, static spherically symmetric geometries with dynamical torsion are determined by the modified double duality equations [10].

We now wish to study stationary, axially symmetric solutions to the modified double duality equations [11]. We also start from the Kerr–de Sitter metric; however, we aim in particular to determine geometries that in the limit of vanishing rotation parameter would go to the static, spherically symmetric solutions given in Ref. [8]. Those reduced equations are given below. The long and tedious algebraic manipulations that led to this system of coupled ordinary differential equations are performed using the exterior calculus package X^TR for REDUCE we developed independently.

2. THE X^TR PACKAGE

Exterior algebra is gaining importance as a computational tool in various branches of theoretical physics. X^TR is a package which enables computations in this formalism in the algebraic simplification language REDUCE 3 [12]. The implementation language has been chosen to be REDUCE because of its widespread use and availability. X^TR has built-in facilities for computations in gravitational and gauge field theories. There already exist some systems which perform computations in the same formalism. Maybe the most well-known system that exists up to this time is EXCALC [13], which is also a package developed for REDUCE and now is distributed along with it.

X^TR is short in code and fast in computation. It uses a different syntax which we believe is more coherent with the general idea behind REDUCE. X^TR should be regarded as an open-code rather than being a computational black-box. As an example, X^TR enables declaration of form-valued functions at user level and provides a handle to access the form-degree(s) of the argument(s) as well as the ability to declare the way of computing the function's form-degree. As another example, X^TR avoids the implicit assumption of summation convention over repeated indices, in order to give flexibility to the user for possible coding of efficient algorithms specific to a problem. It is worthwhile to mention the facility build in X^TR for solving dimensional reduction problems which will not be possible using EXCALC. The technique of dimensional reduction can be described as:

1. There exists an n-dimensional manifold M_n with a topology $M_{n_1} \times M_{n_2}$.
2. The physical quantities (like *connection 1-forms*, *curvature 2-forms*, the *curvature scalar*, etc.) are calculated for M_n.
3. Similar physical quantities are calculated independently for M_{n_1} and M_{n_2}.
4. Using those results, the physical quantities for M_n are tried to be expressed in terms of the physical quantities of M_{n_1} and M_{n_2} only.

The underlying topology of the manifold reflects in the coordinate chart $\mathbf{x}^M$ that it is partitioned as (x^μ, y^m), where the indices run as $\mu = 0, 1, ..., n_1 - 1$ and $m = n_1, ..., n$. This idea of index separation is easily handled with the facilities

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X^TR provides. It is possible to define the dimensional reduction spaces, declare forms to exist in any of them, or to have contributions to both of the spaces. Furthermore, it is also possible to declare indexed forms to have a lifestyle according to their symbolic indices. As would be expected, facilities that define the way the indices shall split while the dimensional reduction is carried out are present.

A manual which will be provided upon request, gives a full description of X^TR, assuming the user has a very limited knowledge of REDUCE and automated symbolic computation. Detailed information on the inner structure of the package, which will aid the advanced user for possible extensions, is also provided.

It is very common that the operations performed in exterior calculus fall into one of the two classes, namely the one in which the operations are antiderivations, where the distribution rule is

$$\triangle(q \wedge r) = \triangle q \wedge r + (-)^{\deg(q)} q \wedge \triangle r,$$

and the one in which the operations are derivations, where the distribution rule is

$$\Box(q \wedge r) = \Box q \wedge r + q \wedge \Box r.$$

X^TR has built in facilities to declare operators with these distribution rules. Also it is possible to have multilinear operations. Among the capabilities of the package are:

- Setting the dimensions and signature of the form space.
- Declaration of variables and REDUCE operators to be forms of any degree.
- Performing the basic operations of exterior calculus, namely:
  - *Exterior (wedge) product.* X^TR knows:
    - The commutation rule, $p \wedge q = (-)^{\deg(p)\deg(q)} \cdot q \wedge p$.
    - A wedge product which contains a form in two different places will cause the product to be zero if this is an odd-degree-form.
    - Any wedge product that sums up to a higher degree than the space dimension in degrees is zero.
    - The wedge product of two zero forms is the ordinary multiplication of them.
  - *Exterior differentiation.*
    - It acts as an antiderivative type of operator over the wedge product.
    - Is nilpotent.
    - It acts as the ordinary differentiation if applied to zero-forms.
    - It is possible to control the expansion into partial derivatives if it is a function that is subject to the operation.
  - *Interior product.*
    - It acts as an antiderivative type of operator over the wedge product.
    - Is nilpotent.
    - It is possible to force the system to leave the interior products, which can not be explicitly calculated, as they are.
  - *Hodge duality.*
    - If the vierbein and the signature are provided then the system is able to carry out the substitution for the hodge operation.
    - It is known how multi-hodges simplify.

In addition to these capabilities, X^TR is also able to carry out the calculations in an orthonormal base, if it is provided with the vierbein/vielbein. The user also has control over the exterior operations by setting some flags of the package ON or OFF.

It is worth mentioning about the facility built in X^TR for "dimensional reduction."

3. EXTERIOR CALCULUS: NOTATION AND CONVENTIONS

The modified double duality equations are

$$*\left(R^{ab} + \frac{\lambda}{2} e^a \wedge e^b\right) = -\left(R^{ab} + \frac{\lambda}{2} e^a \wedge e^b\right)^*, \tag{1}$$

where λ is an arbitrary real parameter; (e^a) are the orthonormal coframes in terms of which the metric

$$g = \eta_{ab} e^a \otimes e^b. \tag{2}$$

We take $\eta_{ab} = \mathrm{diag}(- + + +)$. Together with the connection 1-forms $(\omega^a{}_b)$, the coframes satisfy the structure equations

$$de^a + \omega^a{}_b \wedge e^b = T^a \tag{3}$$

$$d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b = R^a{}_b. \tag{4}$$

Here $T^a = T_{bc,}{}^a e^b \wedge e^c$ are the torsion 2-forms and $R^a{}_b = \frac{1}{2} R_{cd,}{}^a{}_b e^c \wedge e^d$ are the curvature 2-forms of spacetime. A * to the left of a form describes its Hodge dual, defined so that the invariant volume element

$$\begin{aligned} *1 &= e^0 \wedge e^1 \wedge e^2 \wedge e^3 \\ &= \frac{1}{4!} \varepsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge e^d. \end{aligned} \tag{5}$$

A * to the right of a second-rank antisymmetric tensor denotes its dual. For instance,

$$R^*_{ab} = \frac{1}{2!} \varepsilon_{ab}{}^{cd} R_{cd}. \tag{6}$$

The modified double duality equations may also be written in the form

$$*R_{ab} + R^*_{ab} = \lambda *(e_a \wedge e_b). \tag{7}$$

4. STATIONARY, AXISYMMETRIC DOUBLE DUAL CURVATURES

We consider solutions described by the Kerr de Sitter metric [14]

$$\begin{aligned} g = &-\frac{\delta_r}{r^2 + a^2\cos^2\theta}\left[\frac{dt - a\sin^2\theta\, d\varphi}{1 - ka^2/3}\right]^2 \\ &+ (r^2 + a^2\cos^2\theta)\left\{\frac{dr^2}{\delta_r} + \frac{d\theta^2}{1 - (1/3)\,ka^2\cos^2\theta}\right\} \\ &+ \sin^2\theta\left(\frac{1 - (1/3)\,ka^2\cos^2\theta}{r^2 + a^2\cos^2\theta}\right) \\ &\times\left[\frac{a\,dt - (r^2 + a^2)\,d\varphi}{1 - ka^2/3}\right]^2 \end{aligned} \tag{8}$$

expressed in terms of Boyer–Lindquist coordinates (t, r, θ, φ) [15]. For later convenience we define

$$\begin{aligned} (Z_1)^2 &\triangleq 1 - \frac{k}{3}a^2 \\ (Z_2)^2 &\triangleq r^2 + a^2\mu^2 \\ (Z_3)^2 &\triangleq 1 - \frac{k}{3}a^2\mu^2 \\ M^2 &\triangleq 1 - \mu^2 \\ (\delta_r)^2 &\triangleq \frac{k}{3}(r^4 + a^2r^2) + r^2 - 2mr + a^2, \end{aligned}$$

where $\mu = \cos\theta$, m shows the Schwarzschild mass, a is the rotation, and k is a real parameter. We take the orthonormal co-frames

$$\begin{aligned} e^0 &= \frac{\delta_r}{(Z_1)^2 Z_2}\,dt - \frac{aM^2\delta_r}{(Z_1)^2 Z_2}\,d\varphi \\ e^1 &= \frac{aMZ_3}{(Z_1)^2 Z_2}\,dt - \frac{MZ_3(a^2 + r^2)}{(Z_1)^2 Z_2}\,d\varphi \\ e^2 &= \frac{Z_2}{\delta_r}\,dr \\ e^3 &= \frac{Z_2}{MZ_3}\,d\mu. \end{aligned} \tag{9}$$

Then the Levi–Civita connection 1-forms are found as

$$\begin{aligned} \dot\omega^0{}_1 &= \frac{arMZ_3}{(Z_2)^3}\,e^2 + \frac{a\mu\delta_r}{(Z_2)^3}\,e^3 \\ \dot\omega^0{}_2 &= \frac{1}{(Z_2)^5\,\delta_r}\left[a^2\mu^2\left(3a^2\mu^2\delta_r\frac{\partial\delta_r}{\partial r} + ka^2r^3\right.\right. \\ &\qquad \left.- 3a^2r + 3kr^5 + 3r^3\right) + 3mr^4 \\ &\qquad \left.- 3a^2r^3 + kr^7\right]e^0 + \frac{3MZ_1ar}{(Z_2)^3}\,e^1 \\ \dot\omega^0{}_3 &= \frac{a^2\mu MZ_3}{(Z_2)^3}\,e^0 - \frac{a\mu\delta_r}{(Z_2)^3}\,e^1 \\ \dot\omega^1{}_2 &= -\frac{arMZ_3}{(Z_2)^3}\,e^0 + \frac{r\delta_r}{(Z_2)^3}\,e^1 \\ \dot\omega^1{}_3 &= \frac{a\mu\delta_r}{(Z_2)^3}\,e^0 + \frac{\mu}{3MZ_3(Z_2)^3} \\ &\qquad \times\left[ka^2(\mu^2(Z_2)^2 - M^2r^2) - 3(a^2 + r^2)\right]e^1 \\ \dot\omega^2{}_3 &= \frac{a^2\mu MZ_3}{(Z_2)^3}\,e^2 - \frac{r\delta_r}{(Z_2)^3}\,e^3. \end{aligned} \tag{10}$$

We will construct solutions such that the full connection 1-forms are

$$\begin{aligned} \omega_{01} &= \Omega_1 e^2 + \Omega_2 e^3 \\ \omega_{02} &= \Omega_3 e^0 + \Omega_4 e^1 \\ \omega_{03} &= \Omega_5 e^0 + \Omega_6 e^1 \\ \omega_{12} &= \Omega_7 e^0 + \Omega_8 e^1 \\ \omega_{13} &= \Omega_9 e^0 + \Omega_{10} e^1 \\ \omega_{23} &= \Omega_{11} e^2 + \Omega_{12} e^3. \end{aligned} \tag{11}$$

TABLE I

	A	B	$C\cdot D$	$E\cdot F$	$G\cdot H$	$I\cdot J$
S_1	−1	2	8·9	−7·10	6·3	−5·4
S_2	−11	12	8·3	−7·4	−6·9	5·10
S_4	9	−4	−8·1	−7·12	−6·11	5·2
S_5	3	10	−8·11	7·2	6·1	5·12
S_8	−7	−6	4·11	−3·2	−1·10	−12·9
S_9	5	−8	−4·1	−3·12	2·9	−11·10

TABLE II

	A	B	C	D	$E \cdot F$	$G \cdot H$	$I \cdot J$	$K \cdot L$
S_3	4	9	−3	−10	8·2	−7·11	6·12	5·1
S_6	10	−3	−9	4	−8·12	−7·1	6·2	−5·11
S_7	6	−7	−5	8	−4·12	−3·1	2·10	−11·9
S_{10}	−8	−5	7	6	−4·2	3·11	−1·9	−12·10

After manipulating the structure equations we obtain

$$
\begin{aligned}
{}^*R_{01} + R^*_{01} &= S_1 e^0 \wedge e^1 + S_2 e^2 \wedge e^3 \\
{}^*R_{02} + R^*_{02} &= S_3 e^0 \wedge e^2 + S_4 e^0 \wedge e^3 \\
&\quad + S_5 e^1 \wedge e^2 + S_6 e^1 \wedge e^3 \\
{}^*R_{03} + R^*_{03} &= S_7 e^0 \wedge e^2 + S_8 e^0 \wedge e^3 \\
&\quad + S_9 e^1 \wedge e^2 + S_{10} e^1 \wedge e^3 \qquad (12) \\
{}^*R_{12} + R^*_{12} &= -S_{10} e^0 \wedge e^2 + S_9 e^0 \wedge e^3 \\
&\quad - S_8 e^1 \wedge e^2 + S_7 e^1 \wedge e^3 \\
{}^*R_{13} + R^*_{13} &= S_6 e^0 \wedge e^2 - S_5 e^0 \wedge e^3 \\
&\quad + S_4 e^1 \wedge e^2 - S_3 e^1 \wedge e^3 \\
{}^*R_{23} + R^*_{23} &= S_2 e^0 \wedge e^1 - S_1 e^2 \wedge e^3.
\end{aligned}
$$

The functions $S_1, ..., S_{10}$ will be given explicitly below. In terms of these functions the modified double duality equations (7) read

$$
\begin{aligned}
&S_2 = -S_6 = S_9 = \lambda \\
&S_1 = S_3 = S_4 = S_5 = S_7 = S_8 = S_{10} = 0.
\end{aligned} \qquad (13)
$$

These functions are classified in two distinct sets. Six of them, namely S_1, S_2, S_4, S_5, S_8, S_9 are of the generic form:

$$
\begin{aligned}
&\frac{1}{(Z_2)^3}\{MZ_3(a^2\mu + (Z_2)^2\,\partial_\mu)\,\Omega_A \\
&\quad + \delta_r(r + (Z_2)^2\,\partial_r)\,\Omega_B + (Z_2)^3 \\
&\quad \times [\Omega_C\Omega_D + \Omega_E\Omega_F + \Omega_G\Omega_H + \Omega_I\Omega_J]\}.
\end{aligned}
$$

The actual expressions we list in Table I. The first horizontal line in the table means:

$$
\begin{aligned}
S_1 = &\frac{1}{(Z_2)^3}\{-MZ_3(a^2\mu + (Z_2)^2\,\partial_\mu)\,\Omega_1 \\
&+ \delta_r(r + (Z_2)^2\,\partial_r)\,\Omega_2 + (Z_2)^3 \\
&\times [\Omega_8\Omega_9 - \Omega_7\Omega_{10} + \Omega_6\Omega_3 - \Omega_5\Omega_4]\}.
\end{aligned}
$$

The other expressions can be similarly written out.

The second set of functions S_3, S_6, S_7, S_{10} are of the generic form

$$
\begin{aligned}
&\frac{1}{(Z_2)^2\,Z_3}\{MZ_3[-r\delta_r + (Z_2)^2\,\partial_r(\delta_r) + (Z_2)^2\,\delta_r\partial_r]\,\Omega_B \\
&\quad + [\mu(\tfrac{1}{3}kr^2a^2(2\mu^2 - 1) - (r^2 + a^2) \\
&\quad + \tfrac{1}{3}ka^4\mu^4) + M^2(Z_3)^2\,(Z_2)^2\,\partial_\mu]\,\Omega_A \\
&\quad + 2aMZ_3[\mu\delta_r\Omega_C + rMZ_3\Omega_D] \\
&\quad + (Z_2)^3\,Z_3[\Omega_E\Omega_F + \Omega_G\Omega_H + \Omega_I\Omega_J + \Omega_K\Omega_L]\}.
\end{aligned}
$$

The actual expressions are listed in Table II. Their solutions will be discussed elsewhere [16].

5. CONCLUSION

The newly developed exterior calculus package, X^TR, for REDUCE is introduced. Besides its open code design philosophy which enables flexibility in special applications, this package provides additional computational abilities for the dimensional reduction technique that makes it unique. In the future the package will be further developed to include the

- ability to handle Lie algebra valued forms and
- automatic simplification of indexed quantities.

APPENDIX

$X^T R$ extended REDUCE program for the problem described in the preceding sections:

```
% Kerr—de Sitter metric in Boyer—Lindquist coordinates ver 2.1;

COORDINATE TI, PHI, R, MU;
OPERATOR T2, T3, M, DELR;
OPERATOR CONNECTION, DCONNECTION;
ANTISYMMETRIC CONNECTION, DCONNECTION;

% T1 has no R, MU dependency;

LET T2=T2(R, MU),
    T3=T3(MU),
    M=M(MU),
    DELMU=T3(MU)*M(MU),
    DELR=DELR(R);

LET DF(T2(R, MU), R)=R/T2,
    DF(T2(R, MU), MU)=AA**2*MU/T2,
    DF(T3(MU), MU)=(T1**2-1)*MU/T3,
    DF(M(MU), MU)=-MU/M,
    DF(DELR(R), R)=(2/3*K*R**3+(2-T1**2)*R-MASS)/DELR;

INTEGER PROCEDURE ETA(I); %Used for index raising and lowering
  IF I=0 THEN -1 ELSE 1;

PROCEDURE W(A, B)$                    Definition of how to obtain the
   1/2*(ETA(B)*(A .*. D E B) +        connection one-forms.
        ETA(A)*(B .*. D E A) +
        FOR C:=0:3 SUM
        (A .*. (B .*. D E C)*ETA C *E C)$

MATRIX VIERBEIN(4, 4), INVIERBEIN(4, 4);

VIERBEIN(1, 1):= DELR/T1**2/T2 $
VIERBEIN(1, 2):=-AA*M**2*DELR/T1**2/T2 $
VIERBEIN(2, 1):= AA*M*T3/T1**2/T2 $
VIERBEIN(2, 2):=-M*(R**2+AA**2)*T3/T1**2/T2 $
VIERBEIN(3, 3):= T2/DELR $
VIERBEIN(4, 4):= T2/DELMU $

% Below we define the inverse vierbein, in fact, this could be left ;
% to the system by calling GENERATE as GENERATE(T) but the result  ;
% is not as compact as the one below due to the matrix inversion    ;
% routines of REDUCE itself ;

INVIERBEIN(1, 1):= T1**2*(R**2+AA**2)/T2/DELR $
INVIERBEIN(1, 2):=-T1**2*AA*M/T2/T3 $
INVIERBEIN(2, 1):= T1**2*AA/T2*DELR $
INVIERBEIN(2, 2):=-T1**2/T2/T3/M $
INVIERBEIN(3, 3):= DELR/T2 $
INVIERBEIN(4, 4):= DELMU/T2 $

ON DEREXP;

GENERATE( );

ON INBASE; % We want the results in terms of the orthonormal coframes;
```

```
FACTOR E 0&E 1&E 2, E 0&E 1&E 3, E 0&E 2&E 3, E 1&E 2&E 3,
       E 0&E 1, E 0&E 2, E 0&E 3, E 1&E 2, E 1&E 3, E 2&E 3,
       E 0, E 1, E 2, E 3;

OFF UNKINPRD$ % No unknown interior product shall remain;

% First compute the connections and store the result into CONNECTION;

FOR A:=0 : 3 DO
  FOR B:=A+1 : 3 DO
    CONNECTION(A, B):=W(A, B)$

% Now compute the D's of the connections;

FOR A:=0 : 3 DO
  FOR B:=A+1 : 3 DO
    DCONNECTION(A, B):=D W(A, B)$

% Compute the riemann 2-formS;

OPERATOR RIEMANN;
ANTISYMMETRIC RIEMANN;

FOR A:=0 : 3 DO
  FOR B:=A+1 : 3 DO
    RIEMANN(A, B):=DCONNECTION(A, B) +
                   FOR C:=0 : 3 SUM
                     ETA(C)*CONNECTION(A, C) & CONNECTION(C, B);

% PROMETEUS will contain The modified double duality equations;

FOR A:=0 : 3 DO
  FOR B:=A+1 : 3 DO
     PROMETEUS(A, B):=RHODGE RIEMANN(A, B) + # RIEMANN(A, B);

REMCOORDINATE TI, PHI, R, MU; % Just to speed up;

LET
    T1**2         =1-K/3*AA**2,
    T2(R, MU)**2=R**2+AA**2*MU**2,
    T3(MU)**2   =1-K/3*AA**2*MU**2,
    M(MU)**2    =1-MU**2,
    DELR(R)**2  =K/3*(R**4+AA**2*R**2)+R**2-2*MASS*R+AA**2;

% The outputting of the results;

FOR A:=0 : 3 DO FOR B:=A+1 : 3 DO
                  CONNECTION(A, B):=CONNECTION(A, B);
FOR A:=0 : 3 DO FOR B:=A+1 : 3 DO
                  DCONNECTION(A, B):=DCONNECTION(A, B);
FOR A:=0 : 3 DO FOR B:=A+1 : 3 DO
                  RIEMANN(A, B):=RIEMANN(A, B);
FOR A:=0 : 3 DO FOR B:=A+1 : 3 DO
                  PROMETEUS(A, B):=PROMETEUS(A, B);
```


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